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ANNOUNCEMENT

Ken Collier begins, in September, a three-year program at the Starr King School for the Ministry in preparation for the Unitarian ministry. As he will be leaving Southern Illinois University, he is resigning as editor of this Newsletter. His coeditor, Bob Wolf, will take over as sole editor and all future correspondence relating to the Newsletter should be directed to Wolf.

It is with great regret that we make this announcemnt, for not only is the Newsletter Ken's brainchild, it is his energy, organizing ability and ideas which both made the Newsletter a reality and shaped its format. We have profited greatly from our professional association and friendship with Ken. It will be difficult to carry on without him.

Ken's address, beginning September 1, 1976, will be:

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Material addressed to him at the school should also reach him:

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VARIA

Both Nuel Belnap and Mike Dunn have returned to the U. S. after their highly successful stays in Canberra. One of the spin-offs of Mike's trip is in the present issue of the Newsletter.

Nuel Belnap informs us that he and Bob Meyer have decided against a third volume to Entailment. It is hoped to get the manuscript of volume 2 (which Bob is working on at the present time) to the publishers by the end of 1976.

Donald Nute has provided us with a survey of his current researchs in the field of conditional logics. The format of his paper (the first of his two in this issue) seems quite useful and well suited to the Newsletter. We invite other researchers to provide similar overviews of their work.

Michael McRobbie has provided a report on the organizing meeting of the A. A. L. We hope - in future issues - to publish the abstracts of papers read at A. A. L. meetings. If there are other national or local groups who seek an outlet for reports of their meetings, please let us know.

Richard Routley read three (!) papers at the recent Third Latin-American Symposium on Mathematical Logic. The abstracts of these and the other papers read at the symposium (several of which are on paraconsistent logics) will be published in The Journal of Symbolic Logic. The proceedings of the meeting will be published by North-Holland Publishing Company.

In our next issue, we shall publish, among other items, a follow-up by Mr. Almog to his paper in this issue.

Finally, we would like to express our gratitude to the Graduate School of Southern Illinois University, Edwardsville, for subsidizing the printing of this issue of the Newsletter.

The Interrelations Between Fuzzy Logics and the Concept of Relevant Entailment

Introduction

In this paper we want to study some aspects of the relations between fuzzy implication and relevant implication. Lakoff 1974 (p. 260) noticed the close similarity between Łukasiewicz's fuzzy implication and Routley and Meyer's relevant entailment. He suggested that the interrelations between fuzzy logics and relevant logics be studied. Here we shall try to examine one aspect of these logics, namely the conditional 'if... then...'.

Our paper is divided into two sections: one deals with the connection between fuzzy implications and relevant implications. In the second section we examine the concept of relevance. We suggest that it is a fuzzy concept. We suggest that the question as to whether some premises are relevant to the conclusion requires, sometimes, a fuzzy answer, i.e. they are relevant to a certain degree. This obseration permits us to develop a fuzzily relevant entailment, including types of relevant entailment: technically speaking a relevant entailment, strictly speaking a relevant entailment, loosely speaking a relevant entailment. Finally we give an empirical motivation to our suggestion about fuzzily relevant entailment.

I.

First let us define some basic concepts:

(1) an implication $P \rightarrow Q$ is fuzzy iff the derivation of Q from P is true (false) to a certain degree, i.e., the value of the implication is not necessarily bivalent.

(2) an implication is relevant iff the premises are relevant to the conclusion.

The type of fuzzy implication which will be discussed is Łukasiewicz's implication (Rescher 1969, p. 36):

(3) $|P \rightarrow Q| = \begin{cases} 1 & \text{if } |P| \leq |Q| \\ 1 - |P| + |Q| & \text{if } |P| > |Q| \end{cases}$.

The type of relevant implication which will be discussed appears in Routley and Meyer 1973 (p. 206):

(4) $I(A \rightarrow B, a) = T$ iff for b, c in K, such that Rabc and $I(A, b) = T \Rightarrow I(B, c) = T$.

(Read: I = interpretation, Rabc = the worlds (or set-ups) a, b are accessible to c or, as Dunn suggested, 'a, b are compossible relative to c', K = the set of all set-ups, $\rightarrow$ = relevant implication)

Dana Scott in Scott 1973 and in Lakoff 1974 (appendix 1) suggested the following reformulation of (3):

$$V_i(P \rightarrow Q) \equiv \forall\, j, k \text{ such that } i + j \leq k, (V_j(P) \rightarrow V_k(Q)).$$

Scott's valuations are not of degrees of truth (as are Łukasiewicz's or Zadeh's) but of degrees of error. Thus if P deviates from perfect truth to the degree j, and the whole implication deviates from perfect truth to the degree i, the conclusion Q deviates from perfect truth to a degree which is higher than or equal to the sum of i and j. To quote from Scott 1973 (p. 272): "The amount of shift of error between the degree of the hypothesis to that of the conclusion is the measure of error of the whole implication". (Note that 1 will be absolute falsity, 0 absolute truth, and if $P \rightarrow Q$ is true to the degree i, i is the greatest lower bound of the valuations in which $P \rightarrow Q$ is true.)

Now, note that Scott interprets $\leq$ as the accessibility relation (equivalent to R in (4)). Here is the point at which relevant implication

and fuzzy implication have something in common. The conclusion, in both logics, is true on valuations which have valuations, compossibly relative to the valuation of the conclusion, in which both the hypothesis and the implication are true.

Both Scott's interpretation (as presented by Lakoff 1974, p. 257) and Dunn's interpretation of possible worlds allow a uniform treatment of fuzzy and relevant implication. Scott takes a possible world to be a complete and consistent assignment of truth values to every proposition, in other words, a classical two-valued valuation. The accessibility relation is thus a relation between classical valuations. Dunn 1973 takes a possible world to be a truth-functional valuation and a partial valuation, i.e., a possible world is an ordered pair (v, p) where v is a function mapping sentences into the truth values T and F, and p is a submapping of v. (Actually p is like Kripke's relative possibility.) Thus we are able to conclude for both logics that Q is true in valuations to which the compossibly relative valuations in which P is true and $P \rightarrow Q$ is true.

This formulation permits us to compare another requirement of relevant logic with another fuzzy implication. Gödel (cf. Rescher 1969, p. 44) suggested the following fuzzy implication:

$$|P \rightarrow Q| = \begin{cases} 1 & \text{if } |P| \leq |Q| \\ Q & \text{if } |P| > |Q|. \end{cases}$$

Scott (in Lakoff 1974, p. 259, and in Scott 1973, p. 271) suggested that Gödel's implication be reformulated in the following way:

$$V_i(P \rightarrow Q) = \forall j,\ i \leq j\ (V_j(P) \rightarrow V_j(Q)).$$

Here the accessibility relation is reflexive and transitive (an $\underset{\sim}{S4}$ $\leq$). It is interesting to find a corresponding phenomenon in Routley and Meyer 1973 (p. 208, Lemma 1):

$a \prec b$ and $I(A, a) = T \Rightarrow I(A, b) = T$. ($\prec$ = accessiblity relation)

Thus Gödel's fuzzy implication could be reformulated in the following way:

$I(A \rightarrow B, a) = T$, $a \prec b$, then $I(A \rightarrow B, b) = T$.

Since the $\underset{\sim}{\prec}$ has an $\underset{\sim\sim}{S4}$ semantics we can formulate it with Dunn's method of truth value semantics for modal logic. In Dunn 1973 (p. 91), H and H' are interpretations (possible worlds or valuations), C is the accessibility relation and v and p are as before. And so we can translate the Gödel-Scott fuzzy implication into Dunn's semantics by saying:

$V_i(A \rightarrow B)$ and $V_j(A \rightarrow B)$ are compatible iff $i = (v, p)$, $j = (v', p')$ and $v \underset{\sim}{\subseteq} v'$ and $p \underset{\sim}{\subseteq} p'$,

i.e., if C is the alternative relation $iC_{\underset{\sim\sim}{S4}}j$. (See Dunn 1973, p. 91)

THEOREM 1: The following is true of relevant implication and of fuzzy implication: Q (the conclusion) is true in a valuation which is accessible both from the valuation in which P holds and the valuation in which $P \rightarrow Q$ holds.

II.

So far we have presupposed that the concept 'relevance' is a non-fuzzy concept. That means that implicitly we have held that the set of relevant implications is a normal Cantorian set, i.e., for any implication it is necessary that it is relevant or it is non-relevant. But our basic intuitions object to this observation. In a lot of cases there is a dispute over the question "Is it a relevant implication?" and sometimes the answer is "It is relevant to a certain degree". Our 'ideology' will be to claim that 'relevance' is a fuzzy concept and that the set of relevant implications is a fuzzy set, i.e., each implication has a degree of membership in the set. In order to prove our claim, we need to analyse

the whole notion of relevance. Let us begin by asking "What makes an implication relevant and true (relevantly true)?" We suggest that the value of an implication as 'relevant and true' is a function of two factors: truth and relevance. Is it a two dimensional truth-value? In a way, it is. But an assignment of 1 in the relevance dimension is possible only if 1 is the value of the implication in the truth dimension. So the dimension of relevance is not completely independent of the dimension of truth. If $P \rightarrow Q$ is true, then we may pass to the relevance dimension and then the assignment of either 0 or 1 takes place. But if the implication is false, there is no point in saying that it is either relevant or not relevant. When the implication gets 0 in the truth dimension, it automatically receives 0 in the relevance dimension. If truth and relevance were completely independent of each other, then we could have, for an implication $P \rightarrow Q$, a four-valued valuation W which would have been a function of a truth valuation and a relevance valuation. In that case, we could have an ordered pair in which the first assignment is of a truth value, the second of a relevance value. We could have all four combinations:

$\langle 1, 0 \rangle$ TRUE and NOT RELEVANT

$\langle 0, 1 \rangle$ NOT TRUE and RELEVANT

$\langle 1, 1 \rangle$ TRUE and RELEVANT

$\langle 0, 0 \rangle$ NOT TRUE and NOT RELEVANT

But, as we have just said, if $P \rightarrow Q$ is not true, then automatically it is not relevant. So we have only three possible combinations:

$\langle 1, 0 \rangle$ TRUE and NOT RELEVANT

$\langle 1, 1 \rangle$ TRUE and RELEVANT

$\langle 0, 0 \rangle$ FALSE.

So, we might call the assignment of truth-values a quasi-two-dimensional valuation. I.e., there is a two dimensional asignment only after it is verified that P → Q was assigned 1 in the truth dimension. This is a restricted two dimensional assignment. If we look at figure 1, we can say that the move from the T dimension to the R dimension is permitted iff P → Q got 1 in the T dimension.

Note that for the moment we have presupposed that truth and relevance are nonfuzzy concepts and hence the valuations in both dimensions are bivalent. In figure 1 we represent a relevant and true implication by $\underset{\sim}{a}$, a true but not relevant implication by $\underset{\sim}{b}$, and a false implication by $\underset{\sim}{c}$. (Note that $\underset{\sim}{a}$ denotes a square, $\underset{\sim}{b}$ denotes a line and $\underset{\sim}{c}$ denotes a point.)

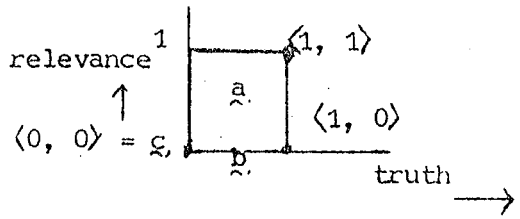

figure 1

THEOREM 2 = the square of an implication is maximal iff it is both true and relevant, i.e., if we have a valuation W which is a combination of valuation T and valuation R, then W(P → Q) = 1 iff (T(P → Q) = 1 and R(P → Q) = 1). So the square of P → Q is maximal iff W(P → Q) = 1; W(P → Q) = S(P → Q). (S = Square)

III.

Now we can drop our requirement concerning "bivalent valuations in the truth dimension". P → Q can be true to a certain degree (just as Łukasiewicz's implication and Gödel's implication are). Hence P → Q can be relevant and fuzzily true (RFT). Now we shall have in the truth dimension a fuzzy valuation. Again W(P → Q) = 1 iff its truth valuation and its

relevance valuation both give to it 1. Suppose $T(P \rightarrow Q) = i$. Then $R(P \rightarrow Q) = j$, $i \succeq j$. Note that there are possible cases in which $P \rightarrow Q$ is true to the degree i but its degree of relevance is 0. Our restriction says that, given $T(P \rightarrow Q) = i$, IF $P \rightarrow Q$ is relevant, it cannot be relevant to a degree higher than i. So if we have a Łukasiewiczian implication in which $|P| \sim |Q|$ and the whole implication has the degree of truth which Q has, then the degree of relevance of this implication is identical to an $|S|$, such that $|Q| \succeq |S|$. But before we pass to fuzzy valuations in the relevance dimension, we want to make some remarks on the dimension of truth.

IV.

So far we assumed that truth is one-dimensional. Herzberger 1974, pp. 78-81, claimed that truth is a function of correspondence to facts and presuppositional security (or bivalence). So if two factors are involved in truth, we can extend our treatment to a three-dimensional valuation of an implication, where the three dimensions are correspondence, bivalence and relevance. So we can have a quasi-three-dimensional treatment, i.e., we have a triple iff at least $P \rightarrow Q$ corresponds to degree 1. (As Herzberger notes, logic is a matter of correspondence, and hence it is sufficient that $P \rightarrow Q$ corresponds, and then it can be relevant. Correspondence is a necessary condition for relevance.)

So if we have three dimensions, we don't have a plane as before (in figure 1). Instead we have a space. So, the volume of $P \rightarrow Q$ is maximal iff $P \rightarrow Q$ has the degree 1 in all three dimensions. Taking C = corresponds, B = bivalence, R = relevance, VO = volume, then $MAX(VO(P \rightarrow Q))$ IFF $C(P \rightarrow Q) = 1$ & $B(P \rightarrow Q) = 1$ & $R(P \rightarrow Q) = 1$.

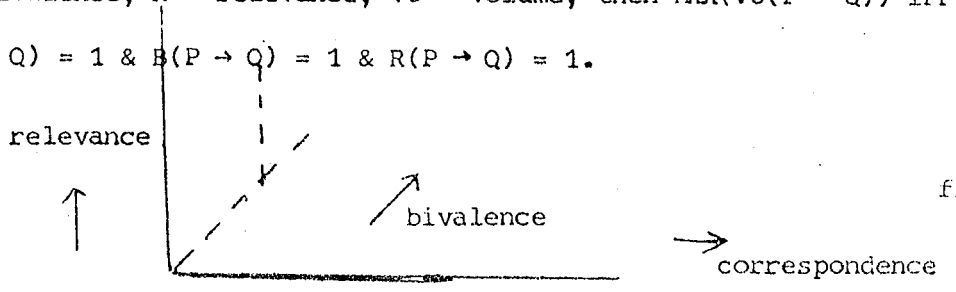

figure 2

V.

Now let us examine the relevance dimension. We want to claim that here too we can have a fuzzy valuation; i.e., relevance is a fuzzy concept and the set of all relevant implications is a fuzzy set in which the implication has a degree of membership. So, just as in the truth dimension, it is possible that we have degrees - degrees of relevance.

It is clear that the relevance valuation will be always equal or smaller than the degree of truth. It is also clear that no matter if we take truth to be one-dimensional or two-dimensional (à la Herzberger) the maximal (highest) value will be assigned to the implication iff it has the maximal values in the components on which we apply a function in order to get the value of the implication as 'relevant and true'.

Our suggestion is motivated by the fact that sometimes it is very hard to define whether the premises are relevant to a conclusion. It is more intuitive to say that there are degrees of relevance, borderline cases of relevance, etc. (See appendix 2 for empirical motivation.) So we get the new notion of fuzzily relevant and fuzzily true (FRFT).

THEOREM 3 = The greater the degree of $P \rightarrow Q$ in the relevance dimension and the greater the degree of $P \rightarrow Q$ in the truth dimension, the greater is the S of $P \rightarrow Q$, i.e., the truth value of $P \rightarrow Q$ as FRFT. (S = square)

VI.

If 'relevant' is a fuzzy predicate, then following Zadeh 1975 and Lakoff 1974, it is characterized by a membership function μ_R. ^{1}(R = relevant) So the membership function for the fuzzy set of relevant implications is equivalent to the absolute value of the predicate 'relevant implication'. Just as the membership function for the fuzzy set of tall men is equivalent

to the absolute value of the predicate 'tall man'. Formally speaking $|RI| = \mu_{RI}$. (RI = relevant implication)

But now let us ask ourselves, "How do we know what is the degree of implication in RI?" How do we know that $\mu_{RI}(P \rightarrow Q) = i$ or j? Well, how do we know whether a robin has an absolute value as bird to the degree i? We have some criteria by which we check 'birdness'. This is the vector value of the predicate 'bird'. We check if it has wings, if it has feathers, etc. So the absolute value is a function of the vector value, formally $|BIRD| = \underset{\sim}{L}||BIRD||$; i.e., the membership function μ_B for the set of birds is itself a function of a k-tuple of criteria which are the vector value of the predicate 'bird'.

The same goes for the set of relevant implications. How do we know what is the degree of membership of a particular implication in this set? Answer: we check the implication in the vector value of the predicate RI; i.e., we have some criteria for relevance and using them we check the particular implication.

Suppose these criteria are G, H, L, so that $||RI|| = \langle \mu_G, \mu_H, \mu_L \rangle$ and so we check the implication in the sets G, H, L. The degree of membership of the implication in the set RI will be a function of its degree of membership in G, H, L; its absolute value as RI will be a function of its vector value as RI. Now as Lakoff 1974, pp. 234-252 (esp. pp. 240-242), notes, among the criteria are: definitional criteria, primary criteria, secondary criteria, etc. These types of criteria help us analyse hedges such as 'technically speaking', 'strictly speaking', 'loosely speaking'. For example: "Nixon is, technically a Quaker" is true because Nixon satisfies definitional criteria for being a Quaker, while it is false that

"Nixon, strictly speaking, is a Quaker" because no primary criteria for being a Quaker (ethical and religious principles) are satisfied by Nixon. Roughly speaking, the following correspondences hold (ignoring some technical points, cf. Lakoff 1974, pp. 240-242):

STRICTLY SPEAKING = PRIMARY CRITERIA (satisfication of ...)

TECHNICALLY SPEAKING = DEFINITIONAL CRITERIA (" ")

LOOSELY SPEAKING = SECONDARY CRITERIA (" ")

Now we can see what will be an implication which is strictly speaking relevant or loosely speaking relevant. In the first case, it will satisfy the most important criteria of relevance and in the second case, it will satisfy only secondary criteria and will be relevant only to a low degree. Formally speaking, we can form three functions f, g, h such that f applies to the vector value of the predicate 'relevant implication' and takes from it only definitional criteria, g takes primary criteria, and h takes secondary criteria. Then we have: (| | = absolute value)

$f(|RI|) = |$TECHINCALLY RI$|$

$g(|RI|) = |$STRICTLY SPEAKING RI$|$

$h(|RI|) = |$LOOSELY SPEAKING RI$|$.

In this way we can give a formal account of the intuitive idea which assumes that there are implications which are technically relevant, strictly speaking relevant and so on. Our formal account does not lack an intuitive base. I think that in everyday life we do have primary, definitional, and secondary criteria and when we are saying that $P \rightarrow Q$ is only loosely speaking relevant, we mean that it satisfies only secondary criteria of relevance. (For another possible treatment of this phenomena with the help of David Lewis' concepts, cf. appendix 1.)

APPENDIX 1

Another possible way to set degrees of relevance might be based on Lewis' notions of 'closeness between worlds' and 'comparative possibility' (Lewis 1974). Suppose we are given the three following implications: $P \rightarrow Q$, $R \rightarrow Q$, $S \rightarrow Q$. Our claim is that: the farther the sphere in which there is a world in which the implication is relevant, the less is the degree of relevance of the implication.

We can use Lewis' connective for comparative possibility $\prec$, which says in $A \prec B$ that A is more possible than B, that some A-world is closer than any B-world. So if $(P \rightarrow Q) \prec (R \rightarrow Q) \prec (S \rightarrow Q)$, then $\mu_{RI}(P \rightarrow Q) = i$, $\mu_{RI}(R \rightarrow Q) = j$, $\mu_{RI}(S \rightarrow Q) = k$, $i < j < k$. Note that if $P \rightarrow Q$ is relevant in the actual world, it has the highest degree of relevance because the actual world is closer to itself than any other world.

The idea of distances between worlds can be explicated in a more metaphorical way: the farther the world, the more bizarre the tale or novel we shall have to tell in order to show the connection between the antecedent and the consequent. Take the following examples:

(1) If there is a revolution in Tibet, the prime minister of Argentina is worried.

(2) If there is a decision procedure for logic, there is no war in the world.

(3) If my dog is ill, Ford will be elected.

Suppose at the actual world, the prime minister of Argentina doesn't care about Tibet. But it is quite possible that in an alternative to the actual world, he will be interested in Tibet (suppose his daughter is traveling in Tibet) and it is not bizarre at all. But (2) and (3) are

bizarre. In order to find a story which will give an explanation, we shall have to invent a very bizarre explanation and we shall use a lot of our imaginative power. So (1) has a degree of relevance which is higher than the degree of (2) or (3).

APPENDIX 2

Empirical motivations

Conditionals in the empirical sciences are sometimes problematic. The connection between the antecedent and the consequent is sometimes neither proved nor disproved. But not only the truth of the conditional is indeterminate. The relevance value is indeterminate, too. In many cases we do not know whether P is relevant to Q. Sometimes it might be the case that P is absolutely relevant to Q; sometimes P is irrelevant to Q. (See below the discussion of Quantum Logic.) Sometimes the relevance of P to Q is a pure matter of chance. The result of this indeterminateness and insufficient information on the relevance of $P \rightarrow Q$ is the need of a postulation of a fuzzy assignment of degrees of relevance. We shall examine two special cases: one in experimental biology and one in quantum physics.

Our 'biological' example is based Buehler 197+. Buehler remarks that there are cases in which $P \rightarrow Q$ gets an intermediate value, when $V(P) = 1$ and $V(Q) = i$, $0 < i < 1$. Hence the relevance of P to Q will be a matter of degree, if $P \rightarrow Q$ is relevant at all.

Buehler doesn't give an example, but think of the research concerning the causes of some cancers. Some materials do not have a catastrophic effect. They have an influence measured in percentages. That means that the conditional $P \rightarrow Q$ (P = the material x was eaten by A, Q = A has a cancer) is true to a degree and hence the relevance of P to Q is a matter of degree. (This uncertainty is a real situation. Scientists do not know exactly which results of the many tests they conducted are relevant to the cancer problem. The same thing happens in Neurology where when one wants to investigate which factors are relevant to a failure in the memory center

and one has only an approximation of the degrees of relevance of the factors.)

Buehler suggests that a possible representation of the validity of an empirical statement A may be the following (a = degree of validity):

$$a = \frac{\text{weighted numbers of reasons for A}}{(\text{weighted numbers of reasons for A}) + (\text{weighted number of reasons against A}).}$$

If this is sound, then we have a lot of cases in which competing theories assign different weights for or against an empirical $P \rightarrow Q$, and hence its degree of relevance will be - again - a matter of degree.

The second example comes from quantum logic. In many situations we have in quantum mechanics a choice negation. $P \oplus P^{\perp}$ is always true, but in many cases neither P nor $P^{\perp}$ are true.[2] So let us try to check whether P implies Q relevantly. Hardegree 1975 suggests that an experiment which tested P be called a P-filter. Suppose we have also a system s in state x. If we have a choice negation, the state a may satisfy neither P nor $P^{\perp}$. The proposed conditions for the truth of the conditional are quite complicated.

For such states the fate of the system s at the P-filter is purely a matter of chance, which incidently is irreducible since the state assumed to be pure is maximally informative (Hardegree 1975, p. 416): "....if a system s in state x passes a P-filter its subsequent state is the projection Ps of x onto subspace P of vectors which satisfy P. Now system s in the resulting Px is certain to pass a Q-filter iff Px satisfies Q....". Thus if P and Q are compatible whenever P passes a P-filter and then a Q-filter it will pass another P-filter.

Formally: $x \in |P > Q|$ iff $Px \in |Q|$. (Remember that Px is like a Stalnaker selection function and so the definition says that $P > Q$ is true if Q is true in the closest possible world in which P obtains.)

Hardegree's conclusion, saying that the quantum conditional has a structure of a counterfactual, strengthens our hypothesis about the fuzziness of relevance, because for $P \rightarrow Q$ to be absolutely relevant, $P \rightarrow Q$ must be absolutely true. "$P \rightarrow Q$ is absolutely true" is sometimes an unacceptable notion in quantum logic because we must presuppose that the P-filter and the Q-filter do not disturb each other's measurements. If s passes a P-filter, s has the property P and if then it passes the Q-filter, it has the property Q and also the property P iff the Q-test didn't 'disturb' the P-test. This condition is not always satisfied. Sometimes it is a pure matter of chance, especially in the case where the behavior of s in the P-filter is uncertain. Then the whole connection between P and Q is in question, because s is supposed to arrive at the Q-filter just after being at the P-filter and, as we said, it doesn't satisfy P or its orthocomplement $P^{\perp}$. Hence the relevance of P to Q (in the case s satisfies neither P nor $P^{\perp}$) is indeterminate. To say that $P \rightarrow Q$ is absolutely relevant is to say that s satisfies always either P or $P^{\perp}$. This is false and thus relevance, when $P \rightarrow Q$ are two propositions in quantum physics ('quantum propositions') is a matter of degree.

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Notes

1 By $\mu_R(x) = i$, we mean that x is in the set R to the degree i.

2 Suppose H is the set of possible states (the 'possible worlds' of Kripke. The supremum of two sub-spaces S and T, S, T $\in$ H, is the smallest subspace which contains S and T + (all resulting limit vectors) + (all linear combinations of states in S and T). So since $(S \cup T) \subset S \oplus T$, there are states in $S \oplus T$ which are neither in S nor in T. Thus it is possible that a vector is not necessarily in P or $P^{\perp}$. It may absent in both. ($P^{\perp}$ is the orthocomplement of P.)

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A VARIATION ON THE BINARY SEMANTICS FOR R-MINGLE*

J. Michael Dunn

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As a preface, let me first advertise three recent papers of mine [5, 6, 7] on a single semantical theme.[1] I cannot forbear mentioning that the semantical results of [5, 6] were explicitly announced in an APA-ASL symposium in New York, December, 1969. Footnotes in [5, 6] of the "back-in-Warsaw-in-1932" genre give more such scholarly information.

The central idea of [5] is that a semantics for the first-degree entailments of E and R may be obtained by looking at valuations as relations (rather than functions) holding between sentences and the two truth values, so that sentences may be "assigned" both the values $\underset{\sim}{T}$ and $\underset{\sim}{F}$, or neither, or of course the customary single truth values.[2] Values of compound sentences are determined inductively in accord with the usual truth-table rules via a double entry book-keeping system, so that e.g., a conjunction may be "true" because both conjuncts are "true", but it may be "false" as well since at least one conjunct is "false." Some care is taken in [5] to motivate these ideas epistemologically and not ontologically - in terms of our beliefs, not the world.

It was also observed in [5] that a semantics for the first-degree entailments of R-Mingle (RM) is obtained by restricting attention to those "ambivalent" valuations that are bivalent, i.e., that always associate each sentence with at least one truth value. In [6] that idea was extended to provide a semantics for the full system RM, and in [7] this was extended once more so as to cover the first-order version of RM, RMQ.[3]

This note is concerned with extending the semantics of [6] in a different direction, so as to encompass a modal variant of RM. With this end in mind, we first summarize the semantics for RM. An RM model structure is an ordered triple (G, K, R), where K is a set, $G \in K$, and R is a reflexive, transitive, and connected relation on K (it does not hurt to add anti-symmetry so as to make R a linear ordering, nor does it hurt to require G to be the R-least element of K). An RM model on such a model structure is a function $\phi(P, H)$, where P ranges over sentential variables and H ranges over members of K. We specify that the possible values of ϕ are $\{\underset{\sim}{T}\}$, $\{\underset{\sim}{F}\}$, and $\{\underset{\sim}{T}, \underset{\sim}{F}\}$, and also require the

Hereditary Condition: for all $H, H' \in K$, if HRH' then $\phi(P, H) \subseteq \phi(P, H')$.

We then extend ϕ to complex sentences inductively as follows:

($\sim\underset{\sim}{T}$) $\underset{\sim}{T} \in \phi(\sim A, H)$ iff $\underset{\sim}{F} \in \phi(A, H)$;

($\sim\underset{\sim}{F}$) $\underset{\sim}{F} \in \phi(\sim A, H)$ iff $\underset{\sim}{T} \in \phi(A, H)$;

($\&\underset{\sim}{T}$) $\underset{\sim}{T} \in \phi(A \,\&\, B, H)$ iff $\underset{\sim}{T} \in \phi(A, H)$ and $\underset{\sim}{T} \in \phi(B, H)$;

($\&\underset{\sim}{F}$) $\underset{\sim}{F} \in \phi(A \,\&\, B, H)$ iff $\underset{\sim}{F} \in \phi(A, H)$ or $\underset{\sim}{F} \in \phi(B, H)$;

($\vee\underset{\sim}{T}$) $\underset{\sim}{T} \in \phi(A \vee B, H)$ iff $\underset{\sim}{T} \in \phi(A, H)$ or $\underset{\sim}{T} \in \phi(B, H)$;

($\vee\underset{\sim}{F}$) $\underset{\sim}{F} \in \phi(A \vee B, H)$ iff $\underset{\sim}{F} \in \phi(A, H)$ and $\underset{\sim}{F} \in \phi(B, H)$;

($\rightarrow\underset{\sim}{T}$) $\underset{\sim}{T} \in \phi(A \rightarrow B, H)$ iff for all $H' \in K$ such that HRH',

(i) $\underset{\sim}{T} \in \phi(A, H')$ only if $\underset{\sim}{T} \in \phi(B, H')$, and

(ii) $\underset{\sim}{F} \in \phi(B, H')$ only if $\underset{\sim}{F} \in \phi(A, H')$;

($\rightarrow\underset{\sim}{F}$) $\underset{\sim}{F} \in \phi(A \rightarrow B, H)$ iff either

(i) $\underset{\sim}{T} \notin \phi(A \rightarrow B, H)$ or

(ii) $\underset{\sim}{T} \in \phi(A, H)$ and $\underset{\sim}{F} \in \phi(B, H)$.

Now we describe syntactically the modal variant of RM that we shall be characterizing semantically. This is the system E plus Mingle (E + M) that results from adding to the system E the axiom scheme

(Mingle) $A \rightarrow . A \rightarrow A$.

There is an awkwardness in names here in that this is not the same as the system E-Mingle (EM) of [1, p.340], which results from adding to E

(Restricted Mingle) $A \to B \to . A \to B \to . A \to B$.

The system EM was first proposed and investigated by Storrs McCall, and like E satisfies the Ackermann matrix that throws out fallacies of modality like Mingle, but unlike E fails to satisfy the more sensitive Maksimova matrix (cf. [1, §22.1.1]).

Semantically, an E + M model is precisely like an RM model except that the Hereditary Condition is dropped as well as the requirement that the accessibility relation be connected.[4]

It is a straightforward exercise to show, modifying the proof in [6] for RM, that E + M is strongly complete and sound with respect to the class of E + M models. But it is a somewhat tedious exercise, so I shall give a few hints about just how the modification is to be carried out.

As for soundness, it is interesting that the only point where the connectedness of the accessibility relation directly affects the arguments of [6] is in showing the Hereditary Lemma (which extends the Hereditary Condition to compound sentences). Thus it is natural that for E + M, where the Hereditary Condition is dropped, that connectedness should go as well. Of course, there are many points in the verification of the RM axioms where the Hereditary Lemma was used. But detailed inspection reveals that there was an essential use at only one point, and that is in the verification of the characteristic R axiom, say,

(Assertion) $A \rightarrow . A \rightarrow B \rightarrow B$.

At other points (with the verification of Restricted Assertion, Suffixing, etc.) it might appear that the Hereditary Lemma is used, but instead only a certain version of it for implication sentences is actually doing the work. This

Positive Hereditary Lemma for Implications.
If HRH', then $\underset{\sim}{T} \in \phi(A \rightarrow B, H)$ implies
$\underset{\sim}{T} \in \phi(A \rightarrow B. H')$,

of course follows immediately from $(\rightarrow \underset{\sim}{T})$. The reader may find suspect the absence of the negative side of the above condition, but the truth is that preservation of falsity along the accessibility relation just plays no necessary role in the verification of axioms.

On the side of completeness, we need to adjust the definition of canonical model from [6] in ways familiar from modal logic. Thus for prime theories T' and T", we define $T' < T''$ iff for all implications $A \rightarrow B$, $(A \rightarrow B) \in T'$ implies $(A \rightarrow B) \in T''$. Where T is a prime E + M theory we define the canonical model structure determined by T to be (G, K, R), where G = T, K = {T': T' is a prime E + M theory and $T < T'$}, and R is $<$. The corresponding canonical model ϕ is then defined so that (i) $\underset{\sim}{T} \in \phi(P, T')$ iff $P \in T'$, and (ii) $\underset{\sim}{F} \in \phi(P, T)$ iff $\sim P \in T'$.

It is obvious that a canonical model structure so defined is in fact an E + M model structure. What needs

arguing is that clauses (i) - (ii) above hold for all compound sentences A. An induction on the complexity of A proceeds as for Lemma 5 of [6], until we come to the case (4) where A = B → C. The argument for (ii) proceeds exactly even there as in [6], the only point of delicacy being that we have

(*) $B \to C \to . \sim B \vee C \to . B \to C$

as a theorem of E + M. The delicacy is that the proof of (*) cited in [6] as T11 of [4] depends ultimately on the proof of

(**) $B \to C \to . C \to . B \to C$,

and the proof of (**) as T9 of [4] involves a use of Permutation which is invalid in E + M (E + M is afterall a subsystem of S4 despite Anderson and Belnap's classification of Mingle as a "fallacy of modality"). But there is another proof of (**) eschewing Permutation, making it a theorem of E + M (the first line is an instance of $X \to . Y \to Z \to . W \to Y \to . X \to . W \to Z$, which is easily proven in FE, the subproof formulation of E):[5]

1.	$B \to . B \to B \to . A \to B \to . B \to . A \to B$ E- theorem	
2.	$B \to . B \to B$	Mingle
3.	$A \to B \to . B \to . A \to B$	1, 2 MP

Turning at last to (i) of Case 4, we first show the following analogue of Lemma 3 of [6], which is a

kind of deduction theorem for E + M:

Lemma 3'. Let $\vec{S}$ be a set of sentences of E + M, all of the form C → D. Then if both $\vec{S}$, A $\vdash_{E+M}$ B and $\vec{S}$, ~B $\vdash_{E+M}$ ~A, then $\vec{S} \vdash_{E+M} A \to B$.

Proof. We first note that the following enthymematic form of the deduction theorem is routinely provable for E + M (t is to be regarded as added to E, cf. [1, pp.342-43]): If $\vec{S}$, A $\vdash_{E+M}$ B, then $\vec{S} \vdash_{E+M}$ t & A → B. Lemma 3' then follows from this by the very same moves as did Lemma 3 in [6] from the corresponding enthymematic form of the deduction theorem there.

Finally, the following analogue of Lemma 4 of [6] obviously suffices to establish (i) in Case 4 of the induction:

Lemma 4'. $\vdash_T A \to B$ iff for all prime E + M theories T' such that $T < T'$, $\vdash_{T'}$ A only if $\vdash_{T'}$ B, and $\vdash_{T'}$ ~B only if $\vdash_{T'}$ ~A.

Proof. Left-to-right is immediate. For right-to-left, suppose $\dashv_T A \to B$. Set $\vec{T} = \{C \to D : \vdash_T C \to D\}$. Clearly $\vec{T} \nvdash_{E+M} A \to B$. But then by Lemma 3', either $\vec{T}$, A $\nvdash_{E+M}$ B or $\vec{T}$, ~B $\nvdash_{E+M}$ ~A. From here the proof proceeds as for Lemma 4 in [6], a standard kind of argument showing that either $\vec{T} \cup \{A\}$ can be extended to a prime E + M theory not containing B, or else $\vec{T} \cup \{\sim B\}$ can be so extended so as to exclude ~A.

In closing I would like to mention a few open problems suggested by the binary semantics for RM and E + M. The first and most interesting is to somehow modify the binary semantics so as to characterize the McCall system EM. One sensitive point in this regard is a curious observation by Belnap and Meyer (communicated privately) that EM has the same first-degree entailments as E and R since Restricted Mingle satisfies a certain implicative extension of M_o (that of the first full paragraph of [1, p.286]). But E + M shares with RM the theorem.

$$A \ \& \ \sim A \ \rightarrow \ B \vee \sim B,$$

as may be shown from the semantics. But the following sketch of a syntactical proof is not wholly without interest, in that it offers some explanation as to just what the difference between Mingle and Restricted Mingle is in the context of E (in the context of R there is of course no difference since the R theorem $A \leftrightarrow (A \rightarrow A \rightarrow A)$ makes all sentences effectively the same as implications):

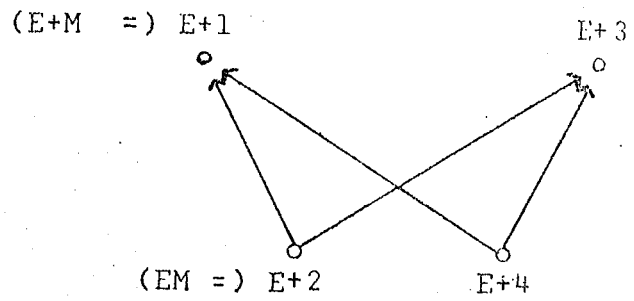
1.	$f \rightarrow . \ f \rightarrow f$	Mingle
2.	$f \rightarrow . \ t \rightarrow t$	1, Contraposition, D. N.
3.	$(t \rightarrow t) \rightarrow t$	t_1 of [1, pp.342-43]
4.	$f \rightarrow t$	2, 3 Transitivity
5.	$A \ \& \ \sim A \rightarrow f$	Routine from t_2 [1, ibid.]
6.	$t \rightarrow B \vee \sim B$	Routine from t_2 [1, ibid.]
7.	$A \ \& \ \sim A \rightarrow B \vee \sim B$	4, 5, 6 Transitivity.

The use of the sentential constants t and f here is of course merely for convenience, since it is well known that they may be added conservatively to E and its extensions. But the main point is that Restricted Mingle would not license step 1.

Finally, I would like to mention that there are other variants of the Mingle axiom that might be of interest. Using '$\vec{A}$' as a metalinguistic variable ranging over implications, some of the more conspicuous variants are

1. $A \to . A \to A$ (Mingle)
2. $\vec{A} \to . \vec{A} \to \vec{A}$ (Restricted Mingle)
3. $A \to . \sim A \to A$
4. $\vec{A} \to . \sim\vec{A} \to \vec{A}$

As a temporary notation, let E + i ($1 \leqslant i \leqslant 4$) be the result of adding axiom scheme i to E. The following containment relations are rather obvious, where the absence of an arrow suggests that a containment relationship fails, but frankly I have not actually verified this in all cases. (Thanks are due to E.P. Martin for querying my original (misdrawn) diagram.)



The axioms 3 and 4 inherit some interest from the well-known fact that necessity may sometimes be defined as $\Box A = \sim A \rightarrow A$ (this is of course not the preferred definition of Anderson and Belnap, cf. [1, §11]).

NOTES

* I wish to gratefully acknowledge that this note was written while I was visiting at the Australian National University on a Fulbright Fellowship from the Australian-American Educational Foundation.

[1] There is also in [5] a syntactical characterization of first-degree entailments using a modification of R.C. Jeffrey's "coupled trees".

[2] This way of making contradictions "both true and false" and tautologies "neither" was discovered independently by Makinson [10] and Fine [8]. It is of course to be compared and contrasted with the Routleys' *-operation [11, 12]. Cf. [9] for an interesting discussion of the two approaches, Also of course van Fraassen [13] has yet another way.

[3] I also might as well advertise some recent related papers of others. Thus Belnap in [2, 3] connects the central idea in [5] of the four values "both T and F," "neither T nor F," "just T," and "just F" with Scott's ideas about approximation lattices in a way that suggests how to extend and complicate the apparatus so as to provide a model for how a computer with a possibly

inconsistent data base might "reason." And Meyer and Belnap in [9] show, among other things, how to use the four-valued approach (in place of the Routleys' *-operation) in providing a semantics for all of R, using, not a binary accessibility relation, not even an "ordinary" Routley-Meyer ternary accessibility relation, but a "four valued" version of this last.

[4] It might be of some interest to axiomatize the models where connectedness is retained. I thought for a while that $(A \to B \to .\ C \to D) \vee (C \to D \to .\ A \to B)$ did the trick for this system "(E + M).3" (the name in analogy with S4.3), but it unfortunately is not valid.

[5] I gratefully recall this proof as having been furnished me by Meyer.

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On $\exists p(\exists qAq \nrightarrow Ap)$

The formula displayed is a tempting addition, schematically, to a relevant logic with propositional quantifiers. However, thanks to some instruction that we all got from Nuel Belnap when he was visiting Canberra, we have learned that it is a temptation which should be resisted. A further diatribe against temptation is the purpose of this note.

First, temptation. What our formula tells us, loosely speaking, is that there is a proposition p such that, if some proposition satisfies the open formula A, then p satisfies A. In the classical logic of propositional quantifiers, or of individual quantifiers (for that matter), this is commonplace. For either nothing satsifies A, whence $\exists qAq \rightarrow Ap$ (and hence its existential generalization) becomes true by falsehood of antecedent, so to speak; or else there is something that satisfies A, whence, for that something p, $\exists qAq \rightarrow Ap$ becomes true by truth of consequent, whence our existentially generalized formula is true.

Reasoning so material, it would seem, scarcely offers a relevant temptation. Following a line, however, which we are all willing at this point to credit generously to Richard Routley, let us call Hilbert to our aid and consider the proposition (?) εpAp, to be understood as a proposition which satisfies A if anything does. Well, then, if something satisfies A, it would seem that a proposition which satisfies A if anything does indeed satisfies A. That is, the line warrants $\exists qA \rightarrow A(\varepsilon rAr)$. And then, existentially generalizing on εrAr, we get $\exists p(\exists qAq \rightarrow Ap)$. Of course, we don't really need Hilbert epsilon operators in the language; the point is that the intuitions--or, at any rate, some intuitions-- justify our initial formula.

Enter Belnap. What Belnap showed us was that, at the level of individual quantification, the putative semantics which Routley proposed to do formal justice to our temptation does not work. But the key Anderson-Belnap counterexample was at the level of propositional quantifiers. Dualizing, it was, near enough, the following instance of the above scheme.

(1) $\neg p(\neg q(q \,\&\, \bar{q}) \rightarrow p \,\&\, \bar{p}$

And I shall now show, by informal argument (which, however, it is easy enough to formalize), how (1) leads, in a system which has it as a theorem, to outright fallacies of relevance.

In fact, suppose that, for some p,

(2) $\neg q(q \,\&\, \bar{q}) \rightarrow p \,\&\, \bar{p}$.

By elementary properties of conjunction in the consequent, we get

(3) $\neg q(q \,\&\, \bar{q}) \rightarrow p$

(4) $\neg q(q \,\&\, \bar{q}) \rightarrow \bar{p}$.

Since anything implies its existential generalization, (3) yields

(5) $q \,\&\, \bar{q} \rightarrow p$.

Working similarly on (4), let's change the variable, getting

(6) $r \,\&\, \bar{r} \rightarrow \bar{p}$.

Contraposing and Demorganizing (6), we get

(7) $p \rightarrow r \vee \bar{r}$.

And applying transitivity to (5) and (7), we get (horrors!)

(8) $q \,\&\, \bar{q} \rightarrow r \vee \bar{r}$.

All other moves being simple, relevantly valid moves, the reader may now convince himself formally that any sane relevant logic with propositional quantifiers that has (1) as a theorem will also have (8) as a theorem. And so, in general, the temptation to embrace $\neg p(\neg qAq \rightarrow Ap)$ as a theorem scheme is a temptation to embrace fallacies of relevance.

Mike Dunn and I, it is true, might argue that the theorem (8) isn't so bad. In fact, we shall defend it in our forthcoming book, R-Mingle and its Rivals. However, (8) ain't relevant, either. And, at the level of propositional quantifiers, any attempt to equip relevant logics with standard prenex normal forming machinery (for example, the equivalence of $\exists p(A \to Bp)$ and $A \to \exists pBp$ will run square into (8). So will the contraposed form $\exists p(Ap \to \forall qAq)$ of our suspect scheme. So, in short, Belnap's intuitions in this area, and his eye for the critical counterexample, are vindicated onece again. That, at any rate, is no surprise.

What happens to the little argument above, the reader may wonder, if we do it all at the level of individual quantifiers, entertaining the scheme $\exists y(\exists xAx \to Ay)$ as a possible addition to, say, EQ? What happens is that we prove

(8*) $Fa \ \& \ \overline{Fa} \to Fb \vee \overline{Fb}$.

At first glance, (8*) doesn't look so bad--at least the shared F carries relevance (and we can show, indeed, that relevance in the sense of sharing predicate variables will be preserved if we yield to our temptation at the ordinary relevant first-order level). At second glance, however, (8*) doesn't look so good; for the individuals a and b may indeed have nothing to do with each other, at which point the shared F ceases to look so impressive.

In many ways, all of this is disappointing. For one thing, having prenex normal forms facilitates completeness proofs (for example, unless I miss my guess, a recent effort to prove completeness by Freeman and Daniels for a relevant logic with, among other stuff, propositional quantifiers is in trouble on the point that leads to (8). But perhaps that effort can be rescued, or maybe it doesn't need rescuing.). And the philosophical

underpinning of Routley's idea is indeed tempting. For the moment, however, what we have are new proofs of fallacies of relevance. Let us hope, accordingly, for a better moment.

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CONDITIONAL LOGIC POTPOURRI

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Professor Wolf has asked me if I would be interested in having abstracts of some recent articles I have written on conditional logics appear in the Newsletter. I would. However, instead of a simply collection of abstracts, I thought it might be more helpful to provide a coherent summary of the series of papers with some explanation of the connections between the various papers. I still think that would be a good idea, but I've decided to settle for this paper instead.

I have been so far the sole perpetrator of seven papers on conditional logic and a collaborator on one other. These include both papers on counterfactual conditionals and papers on causal conditionals. They vary greatly in their degree of formality. Some of them led to a research project entitled 'Nomological conditionals and scientific law' supported by National Science Foundation grant SOC76-08970, and some are results of that project. In fact, preparation of the present report is supported by this grant, and I am very grateful to the Foundation for its continuing support.

My first paper on counterfactuals was [12]. This paper was accepted for publication prior to the appearance of Lewis's [7]. That's just an aside to cover myself against charges of pretention at having adopted the name of Lewis's excellent book for my paper. Other charges will have to be met as they arise. In [12] I reject the theses

(1) $(A > B) \vee (A > \sim B)$

and

(2) $(A \,\&\, B) \supset (A > B)$,

and propose the thesis

(3) $\Box (A \supset B) \supset ((B > C) \supset (\Diamond (A \,\&\, C) \supset (A > C)))$.

Among other things, (3) is a weak enough version of strengthening the antecedent that it can be added to a conditional logic while avoiding

(4) $(A > B) \supset \Box (A \supset B)$.

A possible worlds semantics involving class-selection functions is also developed in [12]. This semantics is essentially the same as that developed independently by Lewis and reported in [6]. Finally, an algebraic semantics for the two logics proposed in this paper is developed and used to produce decidability results.

The primary axes ground in [13] are the adoption of

(5) $((A \vee B) > C) \supset (A > C)$

and the rejection of

(6) $\Box (A \equiv B) \supset ((A > C) \equiv (B > C))$.

These suggestions and the arguments supporting them prompted numerous comments (mostly critical, a few gleefully so) including Loewer [8] and McKay and van Inwagen [9]. I respond to many of these comments in [15]. Here, additional arguments are offered in support of (5) and some alternatives to (6) are suggested. It should be noted that an earlier version of [15] was read at the May, 1976 meetings of the Society for Exact Philosophy at Wayne State University. The substitution principle offered in that paper as an alternative to (6) turns out to still be too strong, resulting along with (5) in (4). The happy knuckle-rapping session that followed the reading of this paper (with myself as guest of honor) pointed out some additional flaws that have

been corrected in [15]. This paper is considerably revised and avoids several faults of that earlier version, but the last word on (5) remains to be said.

[14] is a critical study of Lewis's work on counterfactuals, primarily his [7]. It includes a comparison of his notion of variably-strict conditionals and a notion of non-strict conditionals developed along the lines of the proposals in [13] and [15]. It is also pointed out in [14] that not every model of the class-selection function semantics is equivalent to some model of the system-of-spheres semantics Lewis develops in [6] and [7]. Lewis proves the converse in [6].

Hoping to find a more general semantics than either the class-selection function semantics or the system-of-spheres semantics, a semantics within which these other semantics might be treated as special cases, I developed a version of Kripke's relational semantics in which we have infinitely many possibly distinct accessibility relations, one for each sentence in our language. In this sententially indexed relational semantics, we require for each sentence A an A-accesibility relation. This semantics, however, is not what we were looking for since it turns out to be equivalent to the class-selection function semantics. I then developed a sententially indexed neighborhood semantics along the lines suggested by work by Scott and Montague. In a model of this semantics, each world is assigned a possibly different set of neighborhoods, called A-neighborhoods, for each sentence A in our language. This turns our to be a semantics of the kind being sought. Both of these sententially indexed semantics are developed in [10] where it is also shown that the sententially indexed relational semantics

and the class-selection function semantics are equivalent, and that both of these along with Lewis's system-of-spheres semantics can be treated as special cases of the sententially indexed neighborhood semantics. It is also shown here that another modification of the Scott-Montague semantics due to Hansson and Gärdenfors [4] stands between the system-of-spheres and the sententially indexed neighborhood semantics. Finally, the width (in the sense of Hansson and Gärdenfors [4]) of each of these five semantics is determined.

[11] is a short note in which I adapt results by Fine [2] and Gerson [3] to show that Lewis's system-of-spheres semantics does not have maximal depth in the sense of Hansson and Gärdenfors [4]. More exactly, it is shown that although Lewis's counterfactual logics VC and VCU are both determined by classes of systems-of-spheres, there is a logic L such that $VC \subset L \subset VCU$ which is not determined by any class of systems-of-spheres. This result can be extended and we can show that none of the semantics discussed in [10] have maximal depth.

Turning from counterfactual to causal conditionals, [16] is an attempt to adapt the class-selection function semantics in such a way as to provide an analysis of a strictly causal conditional and to display the relationship between a counterfactual conditional which is based solely on universal or sufficient causation and the ordinary counterfactual conditional without such a restriction. As this paper is appearing in the Newsletter, I will not summarize it here.

Finally, what is certainly the most ambitious project of all from a philosophical point of view, is [1] upon which I collaborated with James Fetzer. In this paper, we develop a calculus which combines

strict implication, counterfactual conditionals, causal conditionals of universal strength and causal conditionals of less than universal strength in one system. A principal goal of this paper is to provide a semantics for statistical statements which incorporates the insights of the single case propensity view developed by Fetzer and others. Kyburg [5] offers a possible worlds semantics for statistical statements which involves looking at infinitely many worlds in each of which infinitely many trials of a certain kind are performed. A statement to the effect that a trial of the relevant kind tends to bring about a certain outcome with strength n is then true just in case in almost all the worlds examined the limit of the ratio of trials with that outcome to the total number of trials approaches n as the number of trials approaches infinity. The 'almost all' locution must itself surely be interpreted in terms of a limit. This semantics incorporates the frequency theorist viewpoint in an obvious sort of way. We suggest instead a semantics in which we look at infinitely many worlds in each of which the same trial occurs. Thus, statistical statements depend for their truth solely on the properties of the single case involved. Our semantics also enjoys the advantage of greater simplicity since it only requires that limits be taken on sequences of worlds while Kyburg's semantics involves taking limits over both sequences of trials within worlds and sequences of worlds. One criticism of the single case propensity view has been that there is no difference between it and the frequency theorist view at the level of formal semantics. [1] demonstrates such a difference.

All of these papers point to a wealth of problems which I hope to

pursue further. I would like to invite correspondence on these or related matters. I would also be happy to provide reprints and/or preprints to interested parties and to receive papers from others working in this area.

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The Logic of Causal Conditionals of Universal Strength*

The relationship between causation and the logic of counterfactual or subjunctive conditional statements has received much attention lately: witness the recent publication of several essays on the subject in [8]. At least one author, David Lewis in [2], has attempted a partial analysis of causal statements within a formal system intended to represent the counterfactual conditional of ordinary discourse. However, as Jaegwon Kim pointed out in [5], ". . . the sort of dependency expressed by counterfactuals is considerably broader than strictly causal dependency and . . . causal dependency is only one among the heterogeneous group of dependency relations that can be expressed by counterfactuals." Conditionals of sufficient cause, or as I will call them, causal conditionals of universal strength, of the form 'P would invariably bring about Q' are, then, a species of counterfactual conditional, i.e., conditionals of the form 'Were P the case, then Q would be the case'. Representing causal conditionals of universal strength by the fork, '$\Rightarrow$', and counterfactual conditionals by the corner, '$>$', we have the principle $(P \Rightarrow Q) \supset (P > Q)$. But any attempt to define causal conditionals solely in terms of counterfactual conditionals does not appear promising. A more likely approach would be to investigate the logical structure of causal conditionals and the relationship between these and counterfactual

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*I would like to thank James Fetzer, without whom this paper would not have been written, for many invaluable comments and suggestions.

conditionals without presupposing that one may be defined in terms of the other. Here I will undertake such an investigation, providing a calculus and a model theoretic semantics for a causal connective of universal strength.

The Counterfactual Calculus C

We begin by assuming a certain axiomatic system as adequately representing the logical structure of counterfactual conditionals as they are used in ordinary discourse. This system, which I will call 'C', is defined by the following inference rules and axioms:

Rules: (1) Modus Ponens (MP),

(2) Deduction within Conditionals (DWC): for any $n \geq 1$,

$$\frac{(P_1 \ \& \ \ldots \ \& \ P_n) \supset Q}{((R > P_1) \ \& \ \ldots \ \& \ (R > P_n)) \supset (R > Q),}$$

Axioms: (1) Truth-functional tautologies,

(2) $P > P$,

(3) $((P > Q) \ \& \ (Q > P)) \supset ((P > R) \equiv (Q > R))$,

(4) $((P > R) \ \& \ (Q > R)) \equiv ((P \vee Q) > R)$,

(5) $(P > Q) \supset (P \supset Q)$.

To this system we add the following definition:

D1: $\Box P =_{df} (\sim P > P)$.

C is at the same time weaker and stronger than the system VC which Lewis supports in [4] and in [3] (where he calls it C1). It is weaker because it lacks Lewis' axiom $(P \ \& \ Q) \supset (P > Q)$ which I rejected in [6] and which Johnathan Bennett convincingly opposes in [1]. It is stronger because I have substituted Axiom (5) for Lewis' weaker $(((P \vee Q) > P) \vee (((P \vee Q) > Q)) \vee (((P \vee Q) > R) \equiv ((P > R) \ \& \ (Q > R)))$. I have offered arguments to support (5) in [7].

The causal axioms to be developed below do not depend in any essential way upon the differences between my C and Lewis' VC. These axioms could be appended to any of the systems of counterfactual logic developed by Lewis in [4] or by Stalnaker and Thomason in [9]. Any changes in the syntax would, of course, require corresponding changes in the model theory.

The Causal Calculus CU

We produce a deductive system CU for the universal strength causal conditional by adding to C axioms for sentences containing the fork and axioms relating the fork to the corner. We have already noted that the causal conditional is a kind of counterfactual conditional. We should also want to maintain the mutual exclusiveness of the logical entailment relation and the nomological causal relations. These observations provide us with our first two causal axioms:

(6) $(P \mathrel{\ddagger} Q) \supset (P > Q)$,

(7) $(P \mathrel{\ddagger} Q) \supset \sim \Box(P \supset Q)$.

The counterfactual connective is non-transitive and non-transposable. The causal connective, on the other hand, has somewhat different properties. It is transitive after a fashion. In fact, the causal connective and the counterfactual in combination enjoy a transitivity of sorts:

(8) $((P \mathrel{\ddagger} Q) \,\&\, (Q > R)) \supset (\Box(P \supset Q) \vee (P \mathrel{\ddagger} R))$.

(8) together with (6) give us a limited transitivity for the fork. The fork is even more restrictive than the corner with respect to transposition since it is a-transposable rather than simply non-transposable:

(9) $(P \mathrel{\ddagger} Q) \supset \sim (\sim Q \mathrel{\ddagger} \sim P)$.

We cannot strengthen (8) to

(8a) $((P \mathrel{\ddagger\!\!>} Q) \ \& \ (Q > R)) \supset (P \mathrel{\ddagger\!\!>} Q)$

without consequences that I find unhappy. One result of (8a) would be

(8b) $((P \mathrel{\ddagger\!\!>} Q) \ \& \ (Q \mathrel{\ddagger\!\!>} P)) \supset (P \mathrel{\ddagger\!\!>} P)$,

again using (6). But then (7) along with (8b) could be used to show

(8c) $\sim((P \mathrel{\ddagger\!\!>} Q) \ \& \ (Q \mathrel{\ddagger\!\!>} P))$.

(8c) may seem reasonable, but the denial of (8c) provides a way of representing co-variance. Identifying co-variance with "co-causation" is admittedly unusual, but it is a useful ploy and causes no formal difficulties that I can discover. It also seems likely that we might find other examples which would lead us to prefer (8) over (8a) although I will not make the attempt here. If we wish to use $(P \mathrel{\ddagger\!\!>} Q) \ \& \ (Q \mathrel{\ddagger\!\!>} P)$ to represent co-variance, this is reason enough to choose the weaker axiom for CU.

(7) gives us the denial of the causal counterpart of (2); it also blocks the causal counterpart of (3), at least in cases of co-variance. Instead of an axiom like (3), we will settle for the substitution of equivalent sentences in causal as well as other contexts. This is accomplished by the following axiom:

(10) $\Box(P \equiv Q) \supset ((P \mathrel{\ddagger\!\!>} R) \equiv (Q \mathrel{\ddagger\!\!>} R))$.

The corresponding result for causal consequents follows from (8). We also add the counterpart of (4):

(11) $((P \mathrel{\ddagger\!\!>} R) \ \& \ (Q \mathrel{\ddagger\!\!>} R)) \equiv ((P \vee Q) \mathrel{\ddagger\!\!>} R)$.

The causal counterpart of (5) is easily derivable from (5) and (6). We also need to preserve the Law of Non-contradiction for causal consequents and to provide a causal axiom to serve as the counterpart of the inference rule DWC:

(12) $(P \mathrel{\ddagger\!\!>} Q) \supset \sim (P \mathrel{\ddagger\!\!>} \tilde{\wedge} Q)$,

(13) $((P \mathrel{\ddagger\!\!>} Q) \ \& \ (P \mathrel{\ddagger\!\!>} R)) \supset (P \mathrel{\ddagger\!\!>} (Q \ \& \ R))$.

Finally, we require one additional axiom relating causal and counterfactual conditionals:

(14) $(P > (Q \not\rightarrow R)) \supset (\Box((P \ \& \ Q) \supset R) \vee ((P \ \& \ Q) \not\rightarrow R))$.

Once again, we cannot use the stronger

(14a) $(P > (Q \not\rightarrow R)) \supset ((P \ \& \ Q) \not\rightarrow R)$,

for then we would have

(14b) $((P \not\rightarrow Q) > (P \not\rightarrow Q)) \supset (((P \not\rightarrow Q) \ \& \ P) \not\rightarrow Q)$.

Axiom (2) then gives us the antecedent of (14b) and MP gives us

(14c) $((P \not\rightarrow Q) \ \& \ P) \not\rightarrow Q$.

But using (5) and (6), we can show that

(14d) $\Box(((P \not\rightarrow Q) \ \& \ P) \supset Q)$,

which together with (7) and (14c) produces a contradiction. These considerations dictate that we accept the weaker (14).

(14) should be recognizable as a version of the familiar exportation equivalence in one direction. We do not want a converse of (14) such as

(15) $((P \ \& \ Q) \not\rightarrow R) \supset (P > (Q \not\rightarrow R))$.

(15) is vulnerable to obvious counter-examples. Suppose P is 'A pushes button B at time t in situation S', Q is 'A is whistling "Dixie" at time t', and R is 'Doorbell D rings at time t*'. Then we might have $(P \ \& \ Q) \not\rightarrow R$, but not have $P > (Q \not\rightarrow R)$, i.e., it would not be the case that were A to push button B at time t in situation S, then it would be the case that A's whistling "Dixie" at time t would cause doorbell D to ring at time t* even though A's pushing button B while whistling "Dixie" in situation S at time t would cause doorbell D to ring at time t*.

One last candidate I will consider for our list of axioms is

(16) $\Box P \supset \sim (P \supset Q)$.

I am inclined to forego this addition. It might be argued, for example, that the mere obtaining of some very obvious analytic statement is sufficient cause for a rational being's believing that statement. It might also be argued that God's existence is necessary and that it is both a necessary and a sufficient cause for the obtaining of some other statement. I may not favor such claims myself, but I would prefer to produce a calculus in which they can at least be formulated without contradiction.

The system CU is defined by the Rules MP and DWC and by Axioms (1) - (14).

Deducibility and Consistency in CU

A sentence P is derivable in CU iff P belongs to the intersection of those sets of sentences which contain all the axioms of CU and which are closed under both MP and DWC. Let CU be the set of all sentences derivable in CU. A sentence P is deducible from a set of sentences S in CU iff P belongs to the intersection of those sets of sentences which contain both CU and S and which are closed under MP. A set of sentences S is CU-consistent iff there is some sentence P such that P is not deducible from S in CU. A set of sentences S is maximally CU-consistent iff S is CU-consistent and for every sentence P, either P is a member of S or $S \cup \{P\}$ is not CU-consistent. It can be shown that if a sentence P belongs to every maximally CU-consistent set of sentences that includes a set of sentences S, then for some finite subset T of S, P is deducible in CU from T.

Model Theory

Our model theory for CU is adapted from Lewis [3]. A c-model is any quadruple $\langle I,[\,],f,g\rangle$ such that:

(c.1) I is a non-empty set,

(c.2) $[\,]$ assigns to each sentence P a subset $[P]$ of I,

(c.3) $[\sim P] = I - [P]$, $[P \,\&\, Q] = [P] \cap [Q]$, and so on for the other truth-functional connectives,

(c.4) f assigns to each i in I and sentence P a subset $f(P,i)$ of I,

(c.5) $[P > Q] = \{i \in I: [Q] \supseteq f(P,i)\}$,

(c.6) $[P] \supseteq f(P,i)$,

(c.7) if $[Q] \supseteq f(P,i)$ and $[P] \supseteq f(Q,i)$, then $f(P,i) = f(Q,i)$,

(c.8) $f(P \vee Q,i) = f(P,i) \cup f(Q,i)$,

(c.9) if $i \in [P]$ then $i \in f(P,i)$,

(c.10) g assigns to each i in I and sentence P a subset $g(P,i)$ of I,

(c.11) $[P \gtrdot Q] = \{i \in I: [Q] \supseteq g(P,i)$ and $i \notin [\sim(P \rightarrow Q) > (P \rightarrow Q)]\}$,

(c.12) $g(P,i) \supseteq f(P,i)$,

(c.13) if $[Q] \supseteq g(P,i)$ then $f(Q,i) \supseteq g(P,i)$,

(c.14) $[P \gtrdot Q] \cap [\sim Q \gtrdot \sim P] = \emptyset$,

(c.15) if $i \in [\sim(P \equiv Q) > (P \equiv Q)]$ then $g(P,i) = g(Q,i)$,

(c.16) $g(P \vee Q,i) = g(P,i) \cup g(Q,i)$,

(c.17) if $[R] \supseteq g(Q,j)$ for each $j \in f(P,i)$, then $[R] \supseteq g(P \,\&\, Q,i)$.

(c.1) - (c.7) are identical with Lewis' (α.0.1) - (α.0.7) in [3], while (c.8) clearly implies Lewis' (α.0.8). Thus, if $\langle I,[\,],f,g\rangle$ is a c-model, $\langle I,[\,],f\rangle$ must be what Lewis calls a standard α-model.

with (c.6) - (c.9) insure the validity$_c$ of instances of Axioms (2) - (5) respectively. Instances of Axioms (6), (8), (11) and (14) are validated$_c$ by (c.11) combined with (c.12), (c.13), (c.16) and (c.17) respectively. The validity$_c$ of instances of Axiom (7) follows from (c.11) and D.1, while that of instances of Axiom (10) follows from (c.11), D.1 and (c.15). For Axiom (9) we require (c.14) to show validity$_c$, while the validity$_c$ of instances of both Axioms (12) and (13) follows from (c.11) and (c.3). From this we derive our soundness result: every sentence derivable in CU is valid$_c$.

Consistency and Completeness of CU

By (c.3), we know that no contradiction is valid$_c$, and we have just shown that every sentence in CU is valid$_c$. Hence, if we can construct a c-model we will be assured that CU is CU-consistent since at least one sentence, in fact all contradictions, are not in CU.

Let $I = \{0\}$. Let C_1 be the set of all sentences containing no occurrances of '>' or '$\not>$'. For $P,Q \in C_1$, let $[P]_1 = I$ if P is a sentential variable, let $[\sim P]_1 = I - [P]_1$, and let $[P \ \& \ Q]_1 = [P]_1 \cap [Q]_1$. For $P \in C_1$, let $f_1(P,0) = g_1(P,0) = [P]_1$. Then for any integer $n > 1$, let C_n be the set of all sentences having at most $n - 1$ occurrences of '>' and '$\not>$'. For $P,Q \in C_n$, let $[P]_n = [P]_{n-1}$ if $P \in C_{n-1}$, let $[\sim P]_n = I - [P]_n$, and let $[P \ \& \ Q]_n = [P]_n \cap [Q]_n$. If $P > Q \in C_n$, let $[P > Q]_n = I$ if $[Q] \supseteq f_{n-1}(P,0)$, and let $[P > Q]_n = \emptyset$ otherwise. If $P \not> Q \in C_n$, let $[P \not> Q]_n = \emptyset$. For $P \in C_n$, let $f_n(P,0) = g(P,0) = [P]_n$. Finally, let $[] = \bigcup_{n=\omega}[]_n$, let $f = \bigcup_{n=\omega} f_n$ and

The intended models are those in which I is the set of all possible worlds and [P] is the set of those worlds in which the sentence P holds. For any sentence P and world i, f picks out those worlds similar enough to i for consideration in determining whether counterfactual conditionals which have P as antecedent are true. Since P would clearly be true if P were true, f only picks out worlds in [P]. For any sentence P and any world i, g picks out all those worlds in which every sentence holds for the obtaining of which the obtaining of P would be a sufficient cause. Since '$\Rightarrow$-' indicates only sufficient causation and not necessary causation, we can have worlds in g(P,i) in which P does not hold.

Some sentences may have no causal consequences. Tautologies may be in this category. We could handle such sentences in one of two ways. First, we could make g assign the empty set to those sentences and any world. This would involve strengthening (c.i2) to read "$g(P,i) \supseteq f(P,i)$ or $g(P,i) = \emptyset$". Alternatively, we can allow g(P,i) to become so large for such a sentence P that only logical consequences of P are true in every world in g(P,i). If we do this, the second condition in the definition of [P $\Rightarrow$ Q] (i.e., (c.ll)) will prevent P from having causal consequences. Obviously, I have elected the latter approach.

Validity and Soundness

A sentence P is <u>valid</u>$_c$ iff for every c-model $\langle I,[\,],f,g\rangle$, [P] = I. We can easily verify each of the following. MP and DWC both preserve validity$_c$. Instances of Axiom (1) are valid$_c$ due to (c.3). (c.5) along

let $g = \bigcup_{n=\omega} g_n$. It is easily verified that $\langle I,[\,],f,g\rangle$ is a c-model. Therefore CU is CU-consistent.

The canonical c-model for CU is the quadruple $\langle I,[\,],f,g\rangle$ where I is the set of all maximally CU-consistent sets of sentences, $[P] = \{i \in I: P \in i\}$, $f(P,i) = \{[Q]: P > Q \in i\}$, and $g(P,i) = \{[Q]: P \Rightarrow Q \in i\}$. We can easily verify that this quadruple is a c-model. One interesting part of this verification is produced by Lewis in [3] where he shows that $[P > Q] \in i$ follows from $[Q] \supseteq f(P,i)$. Here I will look at the corresponding proof that $[P \Rightarrow Q] \in i$ follows from $[Q] \supseteq g(P,i)$ and $i \notin [\sim(P \supset Q) > (P \supset Q)]$. By hypothesis, Q belongs to every member of $g(P,i)$, i.e., to every maximally CU-consistent set of sentences that includes $\{R: P \Rightarrow R \in i\}$. Thus, Q is deducible from some finite subset of $\{R: P \Rightarrow R \in i\}$. Then either Q is a theorem of CU or there are $R_1,\ldots, R_n \in \{R: P \Rightarrow R \in i\}$ such that $(R_1 \,\&\, \ldots \,\&\, R_n) \supset Q$ is a theorem of CU. If Q is a theorem of CU, then $P \supset Q$ is also a theorem of CU, $[P \supset Q] = I$, $[P \supset Q] \supseteq f(\sim(P \supset Q),i)$, and $i \in [\sim(P \supset Q) > (P \supset Q)]$ contrary to hypothesis. Alternatively, since $R_1,\ldots, R_n \in \{R: P \Rightarrow R \in i\}$, $P \Rightarrow R_1,\ldots,P \Rightarrow R_n$ are all in i. But since i is maximally CU-consistent, we have $P \Rightarrow (R_1 \,\&\, \ldots \,\&\, R_n)$ by Axiom (13) and MP. We can easily show that since $(R_1 \,\&\, \ldots \,\&\, R_n) \supset Q$ is a theorem of CU, so is $(R_1 \,\&\, \ldots \,\&\, R_n) > Q$. Therefore, $i \in [(R_1 \,\&\, \ldots \,\&\, R_n) > Q]$. But then since i is maximally CU-consistent, we have $i \in [\Box(P \supset Q) \vee (P \Rightarrow Q)]$ by Axiom (8) and MP. But by D1, $[\Box(P \supset Q) \vee (P \Rightarrow Q)] = [\sim(P \supset Q) > (P \supset Q)] \cup [P \Rightarrow Q]$. By hypothesis, $i \notin [\sim(P \supset Q) > (P \supset Q)]$, so $i \in [P \Rightarrow Q]$.

By a familiar line of argument we can conclude that if a sentence P is valid in every c-model, then it is valid in the canonical c-model and hence is a member of every maximally CU-consistent set of sentences and is derivable in CU. Combining this with our soundness result, we conclude that all and only theorems of CU are valid_c.

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Apologetic Postscript

Unfortunately, the following are derivable in CU:

(17) $(P > Q) \equiv \Box (P \supset Q)$

(18) $(P \mathrel{\not>} Q) \supset \Box (P \supset Q)$

Given (7), we use (18) to show

(19) $\sim (P \mathrel{\not>} Q)$.

CU is consistent, but terribly uninteresting. An examination of the consistency proof for CU will reveal that the c*model constructed in the proof in fact validates (19).

(17), (18) and (19) can be avoided in one of two ways. First, we can weaken (4) and (11). This is the course Lewis follows in [4]. Second, we can weaken (3) and (10), giving up closure under substitution of provable equivalents. A version of this paper was read at the meetings of the Association for Symbolic Logic, New York City, December, 1975, in which these latter two axioms were abandoned altogether. Possible alternatives to (3) are discussed in my 'Simplification and Substitution of Counterfactual Antecedents', Philosophia, forthcoming, an early version of which was read at the meetings of the Society for Exact Philosophy, Detroit, May, 1976.

I make this paper public at this time despite its obvious and serious flaws because I intend to work on a longer and (hopefully) better paper on the same topic in the near future. I hope that this early version will prompt some comments from readers. I am not only concerned about the problems listed in this postscript, but also about other features of CU such as the rejection of (8a) and (8c) and the occurrence of nested $\not>$'s in the sentences of CU.

Finally, the footnote in which I thank James Fetzer is totally inadequate. Fetzer's 'The Likeness of Lawlikeness,' Boston Studies in the

Philosophy of Science, Vol. 32, R. Cohen and A. Michalos, eds. (Dordrecht: D. Reidel Publishing Company, 1976) is the origin of many of the ideas and notational conventions used here, including the notion of causal conditionals of universal strength. During the past eighteen months, Fetzer has also influenced my thinking greatly as we have worked together on a more ambitious paper entitled 'Syntax, Semantics, and Ontology: A Probabilistic Causal Calculus.' In fact, this paper is a preparatory piece for both this joint paper and the longer version of this paper predicted eralier.

My research was supported in part by a National Science Foundation Grant, SOC76-08970.

Australasian Association for Logic

At a conference of logicians that was sponsored by the Research School of Social Sciences, The Australian National University, and which was held in Canberra, Australia, from 22 to 24 August 1975, it was decided to formally reconstitute the Australasian Association for Logic (A. A. L.), which had not met for several years. The meeting elected a new executive which will serve until the next general meeting of the A.A.L. in August 1976. Mr. F. R. Routley of the Department of Philosophy, Research School of Social Sciences, The Australian National University, was elected President, and Mr. M. A. McRobbie of the Department of Philosophy, Research School of Social Sciences, The Australian National University, was elected Secretary.

The object of the A. A. L. is to further studies in logic, in particular in formal logic, history of logic, foundations of mathematics, philosophy of mathematics and philosophy of logic, and the application of formal logic to problems in philosophy, in the social sciences and natural sciences, and in technology. In order to advance these aims, the A. A. L. will be holding an annual conference. The 1976 annual conference of the A. A. L. will be held at The Institute of Advanced Studies, The Australian National University, Canberra, Australia, from Thursday 12 August 1976, until Saturday 14 August 1976.

Membership of the A. A. L. is open to any person wo is interested in the study of logic, and the furtherance of its study. Any person who wishes to join the A. A. L. and thereby receive notices of the A. A. L.'s activities and conferences should contact the Secretary at the following address:

Mr. M. A. McRobbie
Secretary,
Australasian Association for Logic,
Department of Philosophy,
Research School of Social Science,
The Australian National University,
Canberra, A. C. T. 2600, Australia.

Logicians and other interested persons from outside Australia or New Zealand who will be in Australia or New Zealand at the same time as the A. A. L.'s annual conference is being held and who wish to attend or participate in the conference should contact the Secretary at the above address.

5. λ- Scales, κ-Souslin Sets and a New Definition of Analytic Sets

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6. Necessary Existential Dependence

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7. Reply to my Critics

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8. Decision Problems of Voting

Dr. C.T. Rogers,
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9. Semantic Tableaux for First-Degree Relevant Implications

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1975 CONFERENCE OF THE AUSTRALASIAN ASSOCIATION FOR LOGIC

After not having met for a number of years, the Australasian Association for Logic held a highly successful conference at the Research School for Social Sciences, Australian National University from August 22 to August 24, 1975. The Research School also co-sponsored the conference. Nine papers were given on a wide range of subjects and the titles of the papers with their respective authors are given below. A business meeting was held at 8.30 p.m. on August 22 and a new executive was elected, consisting of Mr. F.R. Routley (President) and Mr. M.A. McRobbie (Secretary) The 1976 conference of the AAL will again be held at the Research School for Social Sciences, Australian National University, and is again generously co-sponsored by the Research School as well as the Association for Symbolic Logic. The conference will be held from Friday, October 8, until Sunday, October 10.

M.A. McRobbie

1. The Naive Conception of Set

Dr. M. McDermott,
Department of Traditional and Modern Philosophy,
University of Sydney,
SYDNEY, N.S.W. 2006,
AUSTRALIA.

2. An Inconsistent System in Combinatory Logic

Dr. M.W. Bunder,
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3. Bizarre Dialectics

Mr. J.D. MacKenzie,
Department of Education,
Capricornia Institute of Advanced Education,
ROCKHAMPTON, Q'ld, 4700,
AUSTRALIA.

4. Skolem's Paradox and Skolemism

Dr. R.T. Brady,
Department of Philosophy,
La Trobe University,
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Since the Newsletter is concerned with making results known, details of proofs should, in general, be omitted. We want to make people aware of what's going on, but we do not wish to replace the recognized journals.

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Articles reporting purely technical results and articles raising philosophical considerations will be equally welcomed.

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